DesignCon® 2014

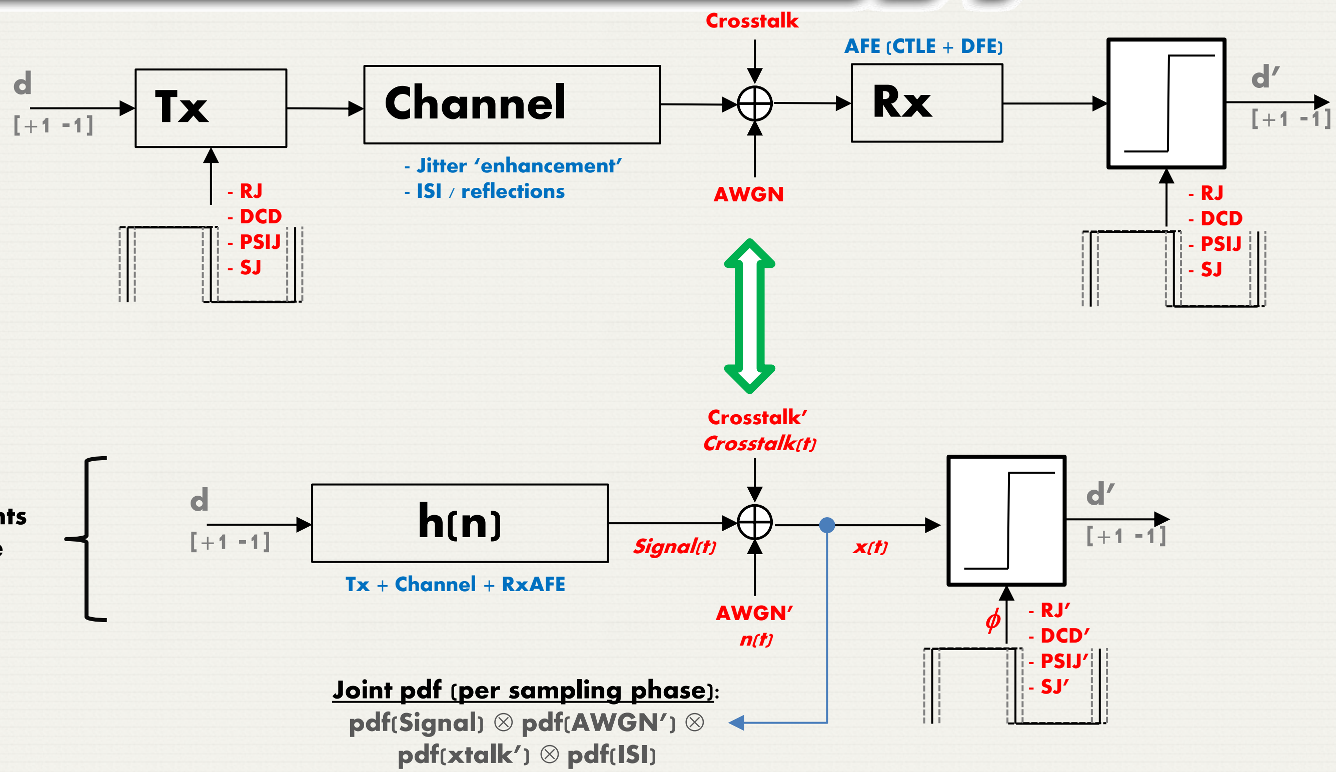
Method for analytically calculating BER (bit error rate) in presence of non-linearity

Gaurav Malhotra
Xilinx

Outline

- Review existing methodology for calculating BER based on linear system analysis.
  - Link model with ISI, Crosstalk, Jitter, Noise.
- Model of nonlinearity based on power series.
- Modification of PDF in presence of nonlinearity.
- BER results for a typical high speed link.
- Link model with multiple linear & NL blocks.

Linear system : Link Model

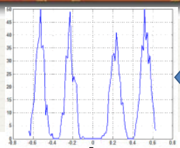


Equivalent linear model:

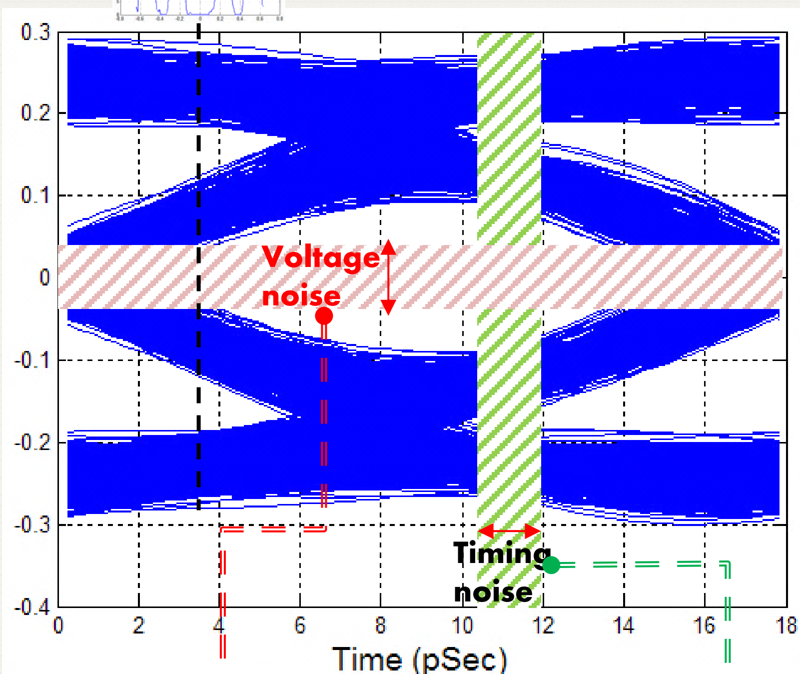
- Signal and impairments can be referred to the slicer input.

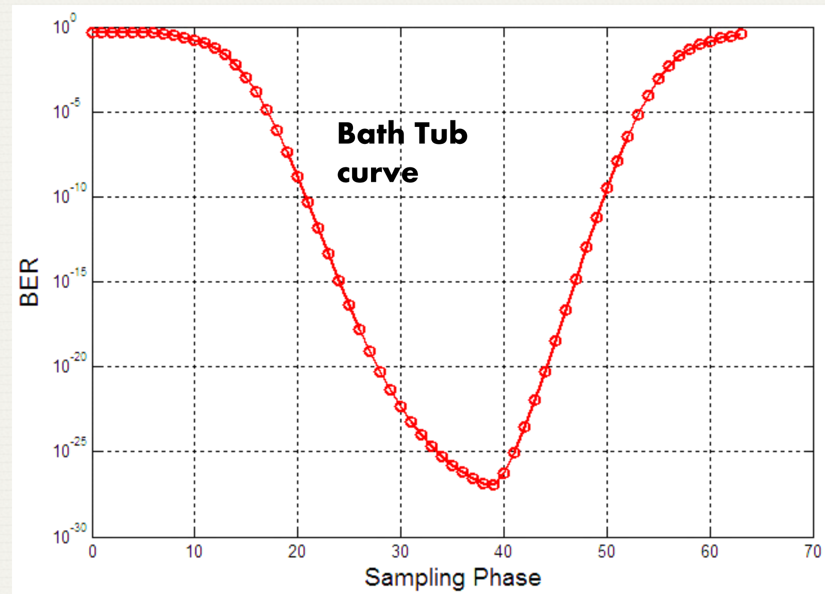
- Objective is to determine joint PDF $F_X(x)$ of signal + impairments [*x(t)*] at the decision point.
- $BER_{\Delta k} = P(error) = \sum_k P(error|d_k)P(d_k)$, where $P(error|1) = \int_{-\infty}^{SS} F_X(x|1)$ S
- Taking timing jitter into account : $BER = \sum_k BER_{\Delta k} \quad F_\Delta(k)$

LTI Systems: Link BER methodology



Joint pdf for Each phase





Joint pdf (per sampling phase): pdf(AWGN) ⊗ pdf(xtalk) ⊗ pdf(ISI)

Joint pdf : pdf(RJ) ⊗ pdf(SJ) ⊗ ..

Conditional pdf

$$BER = \sum_{k} BER_{\Delta k} \quad F_{\Delta}(k)$$

Recap goal

- If we can accurately determine the probability distribution at the decision point, we can calculate BER.
  - $BER_{\Delta k} = P(error) = \sum_k P(error|d_k)P(d_k)$, where

    $P(error|1) = \int_{-\infty}^{SS} F_X(x|1)$ S
  - Taking timing jitter into account: $BER = \sum_k BER_{\Delta k} \;\; F_{\Delta}(k)$

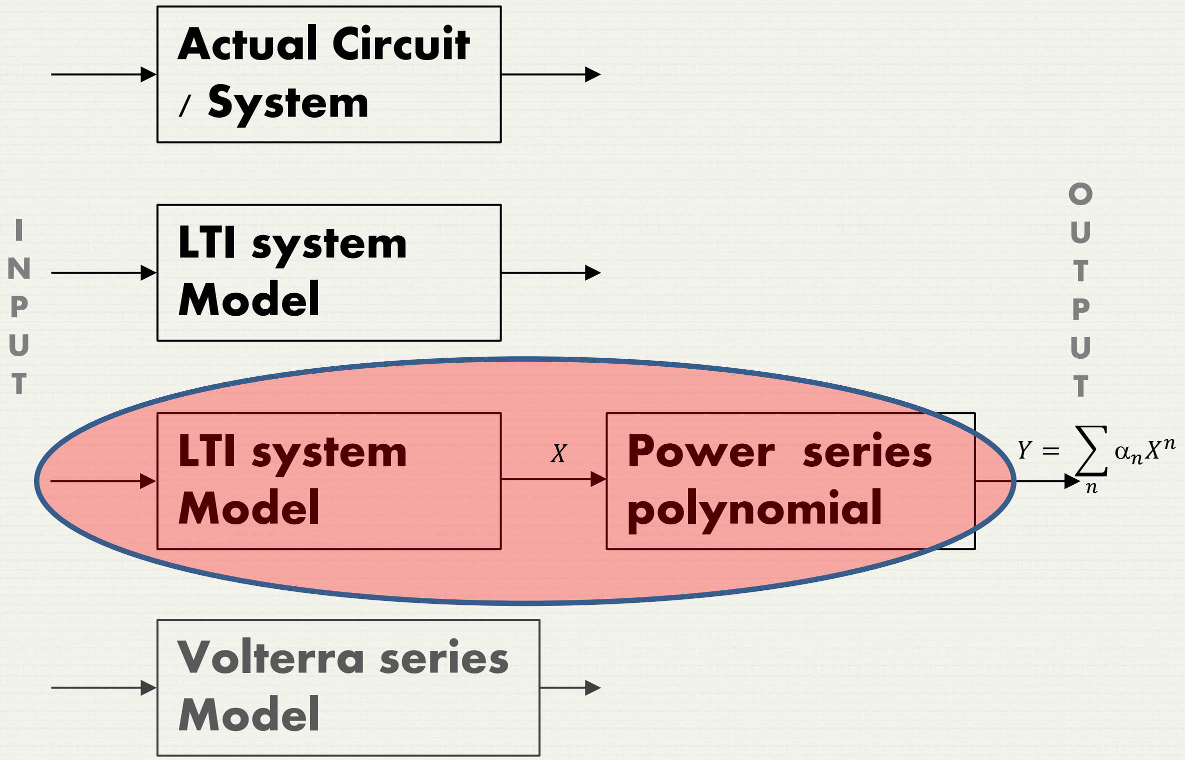
- GOAL: to determine PDF (overall/joint including all impairments AND nonlinearity) at the decision point.

Modeling of nonlinearity

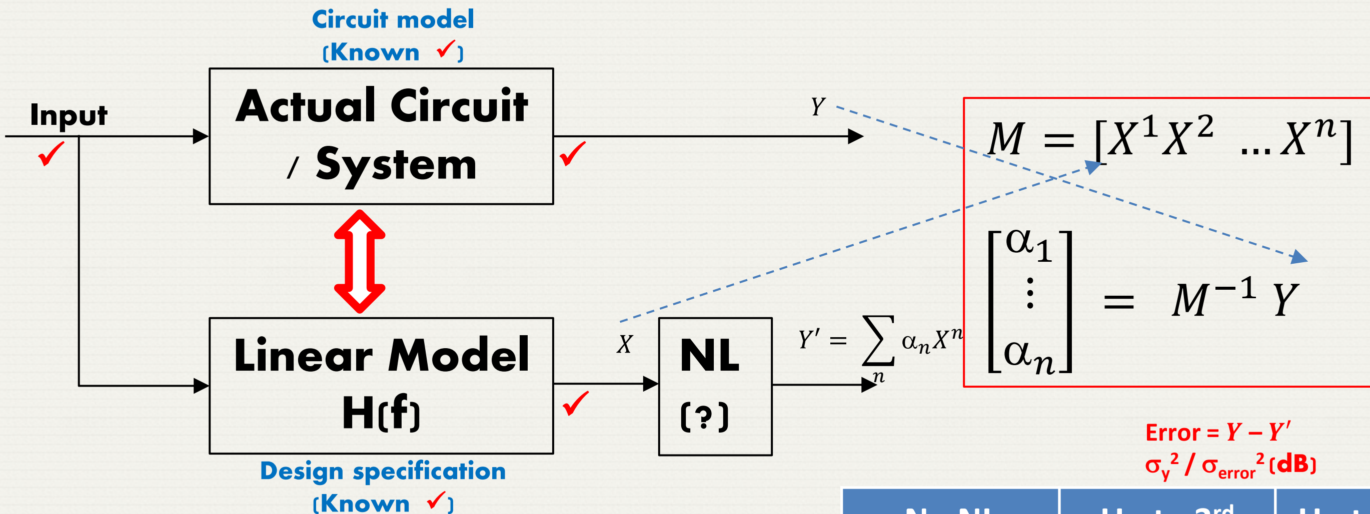
- Common model of NL:

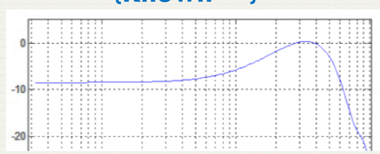
$$Y = \sum_n \alpha_n X^n$$

- Observed to be very close to real circuits.



Modeling of nonlinearity





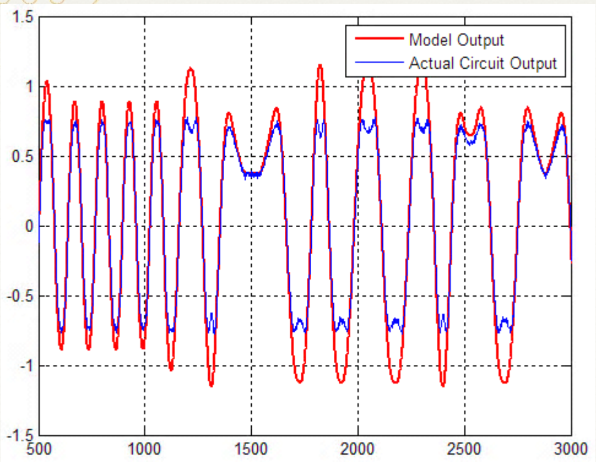
Error = $Y - Y'$

$\sigma_y^2 / \sigma_{error}^2$ (dB)

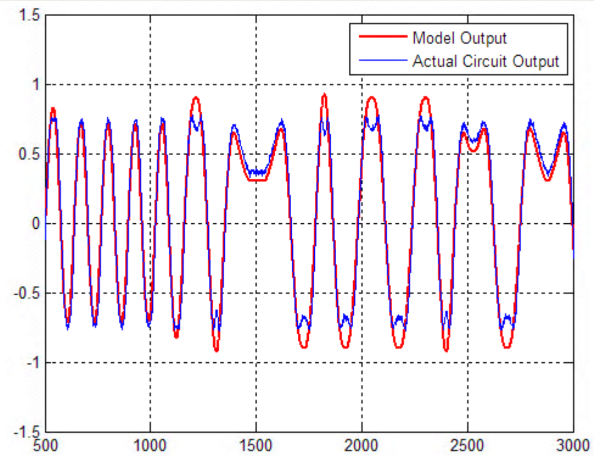
No NL modeling	Up to 3rd order	Up to 5th order	Up to 7th order
11	23	46	51

- Design specification (say pole-zero model) is known.
- Input, X, Y , Y' are time domain signals. Only NL terms $\{\alpha_n\}$ are unknown.
- Matrix inversion (zero forcing) though not optimum, but gives a good estimate of NL terms.

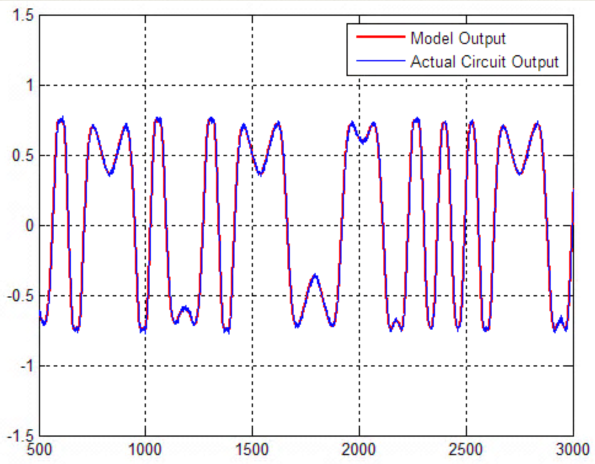
Modeling of nonlinearity



No NL modeling

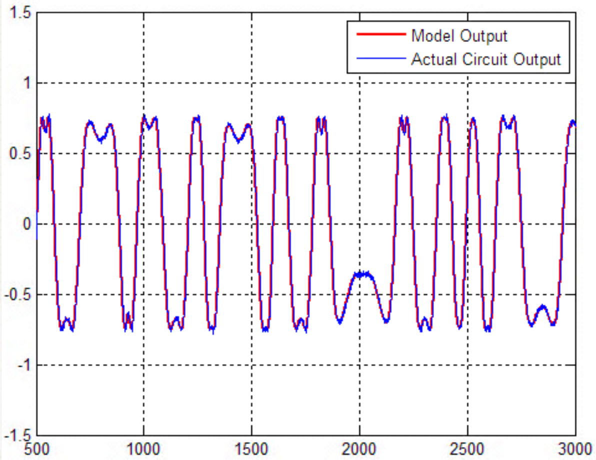


Up to 3rd order



Up to 5th order

No NL modeling	Up to 3rd order	Up to 5th order	Up to 7th order
11	23	46	51



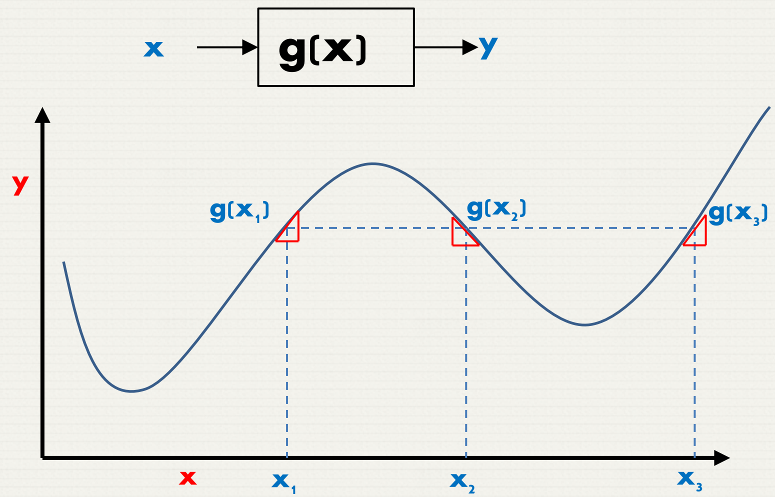
Up to 7th order

- Adding higher order terms in estimation reduces error in modeling due to NL.

Modification of PDF in presence of NL

- Let y = g(x) represent the output of a non-linear function whose input is x.
- The PDF of Y, $F_Y(y)$ can be determined in terms of PDF of X as:

[*Probability, Random variables and Stochastic Processes: Athanasios Papoulis, Section 5-2*]



pdf of x = $F_X(x)$

pdf of y =

$$F_Y(y) = \frac{F_X(x_1)}{|g'(x_1)|} + \frac{F_X(x_2)}{|g'(x_2)|} + \cdots \frac{F_X(x_n)}{|g'(x_n)|}$$

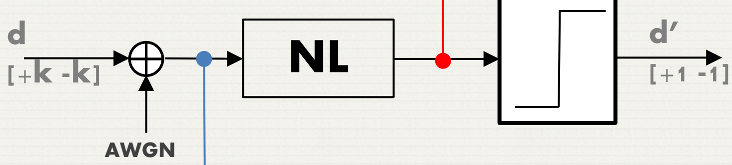
$Simplification\ for\ monotonic\ functions:$

$$F_Y(y) = |\frac{dx}{dy}|\, F_X(x)$$

Modification of PDF : AWGN Example

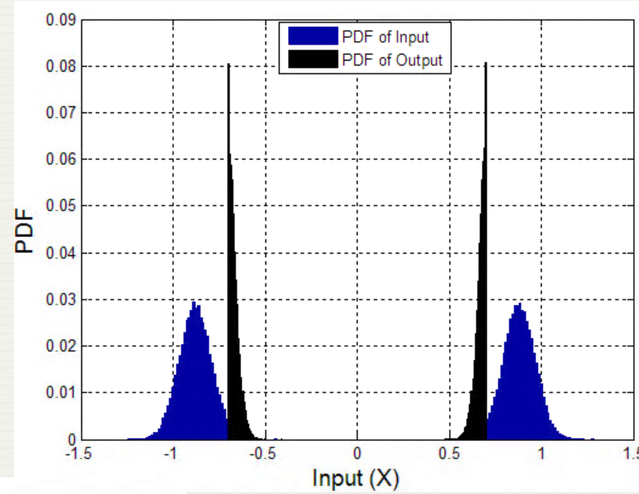
Y= X + αX³

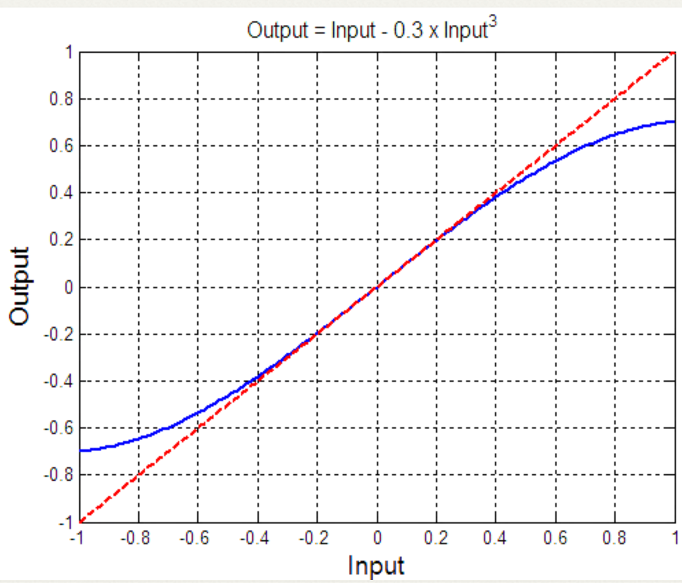
$F_Y(y) = |\frac{dx}{dy}|\, F_X(x)$

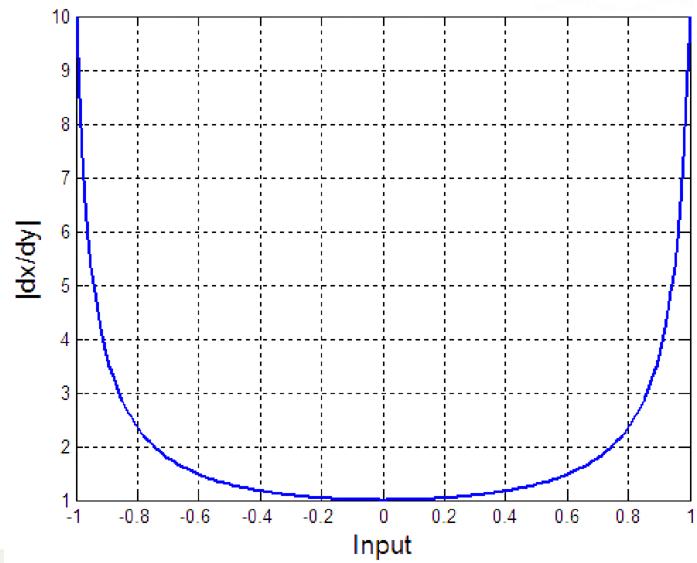


X= Signal +AWGN

$F_X(x)$ = **pdf(Signal) ⊗ pdf(AWGN)**

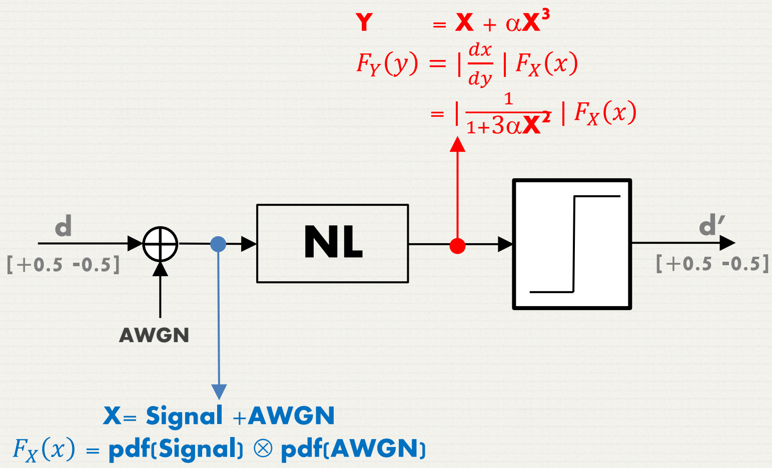


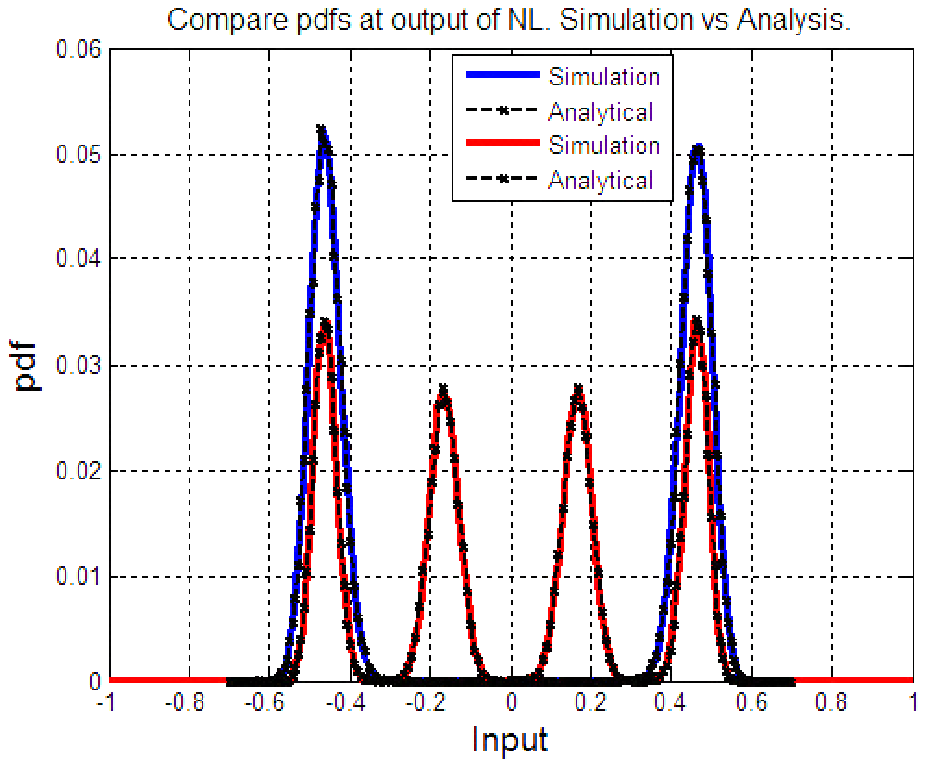




Note the 'warping' of PDF in accordance with $|\frac{dx}{dy}|$

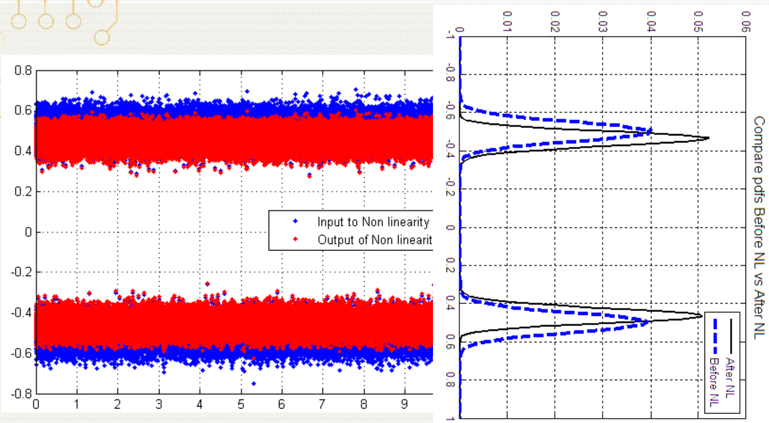
AWGN Example : Simulation VS Analysis

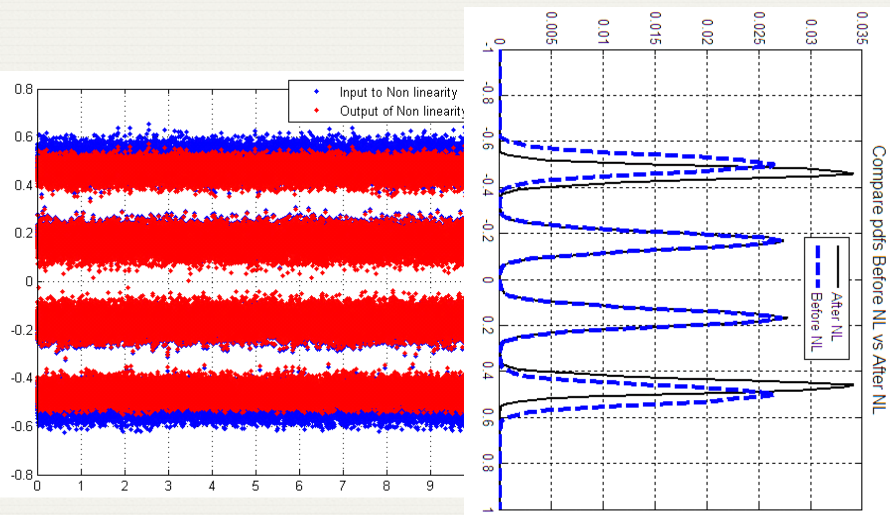




- PDF can be obtained analytically or by running a bit-by-bit simulation.
- Both methods give the same result.
- Analytically computing BER is much faster. This is the method we will adopt for this presentation.

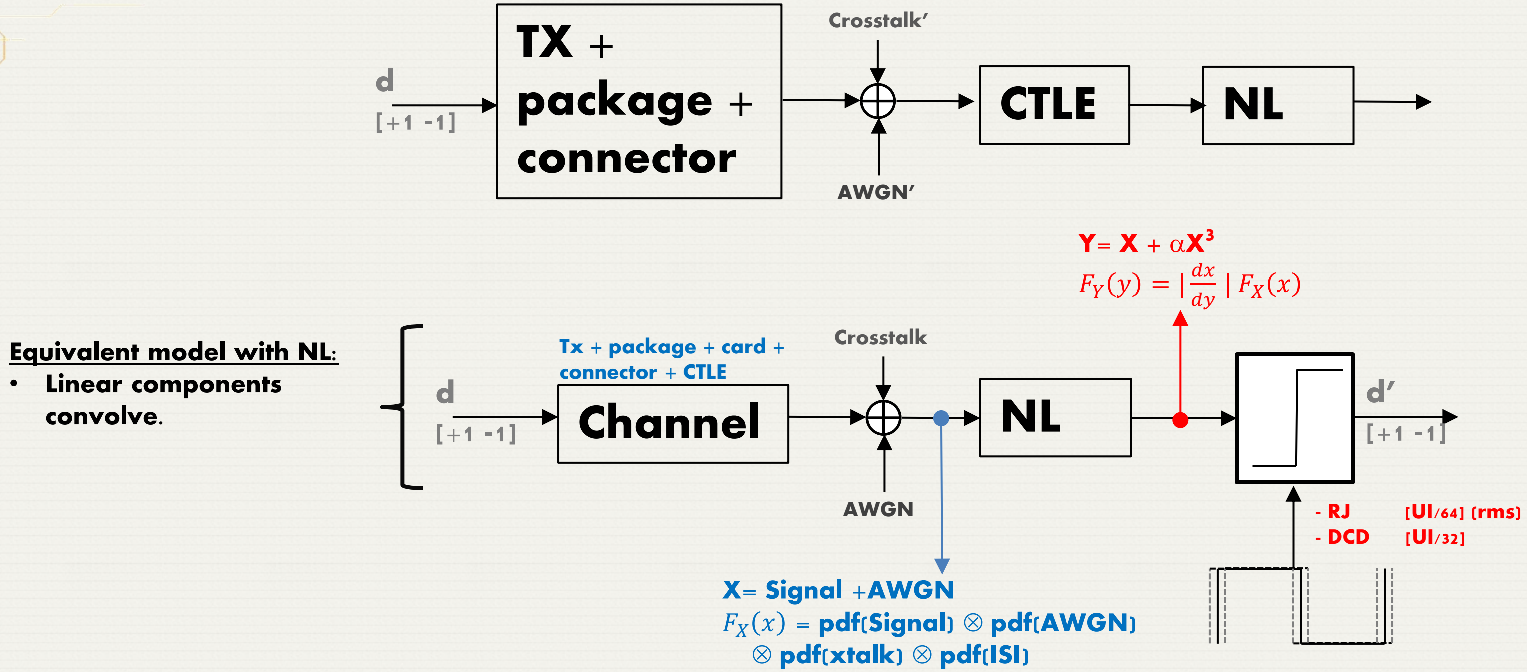
AWGN Example : PAM2 VS PAM4





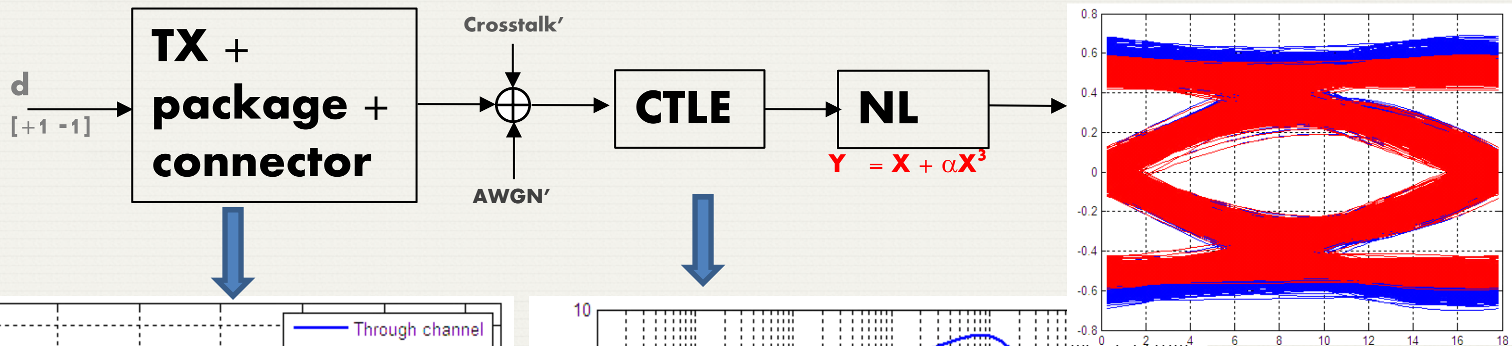
- In general we expect higher order modulations to suffer more from NL.
- Outer points in constellation dominate BER.
- *** Detection rule may be modified to take advantage of (known) non-linearity. This paper assumes that same detection rule (minimum distance) as is used for linear system analysis is used for calculating BER in presence of non-linearity.

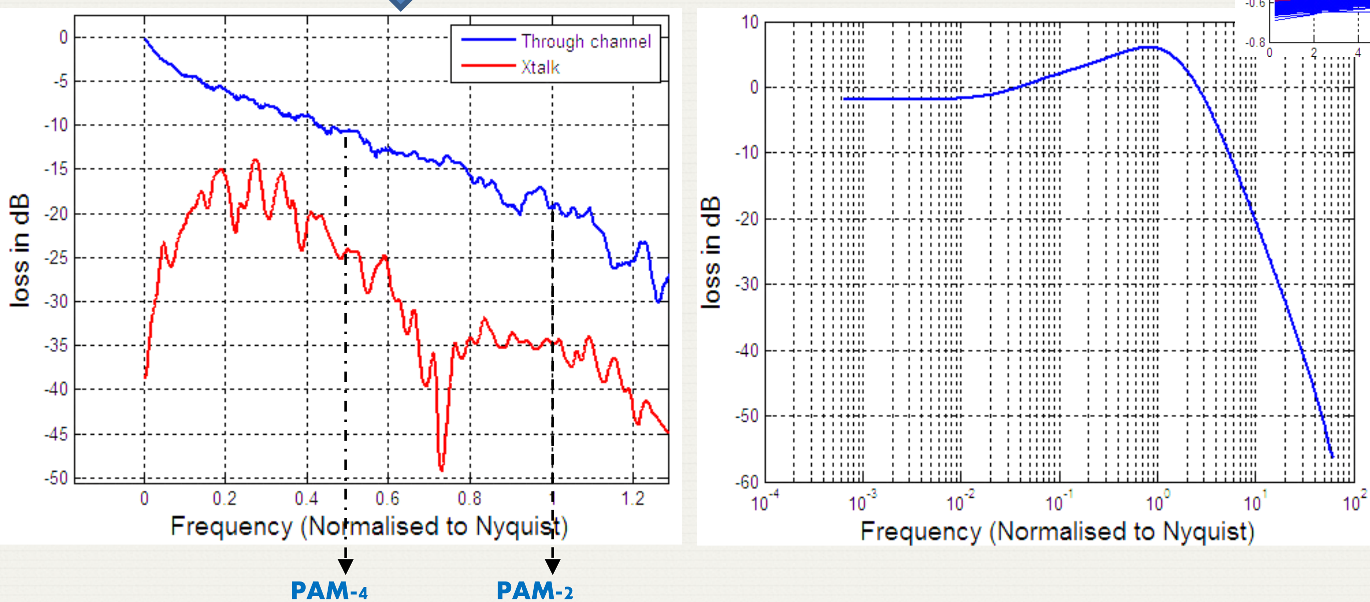
Link Model: typical high speed link



- CTLE (Analog front end) is a significant source of NL.
- α = -0.3; Output = Input – 0.3 * Input^3
- CTLE output referred Xtalk : Xtalk_Out(f) = Xtalk_In(f) * CTLE(f)

Link Model: typical high speed link





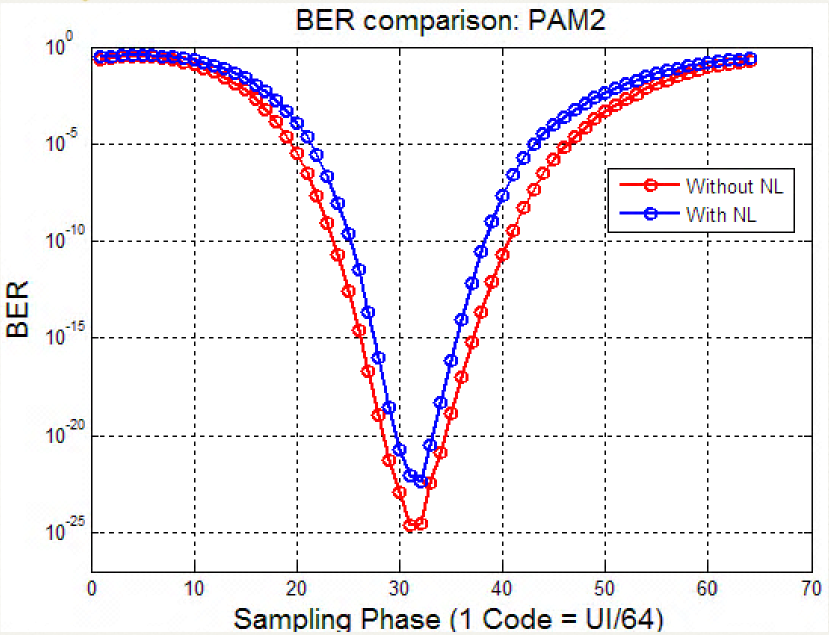
BASELINE: PAM2 VS PAM4

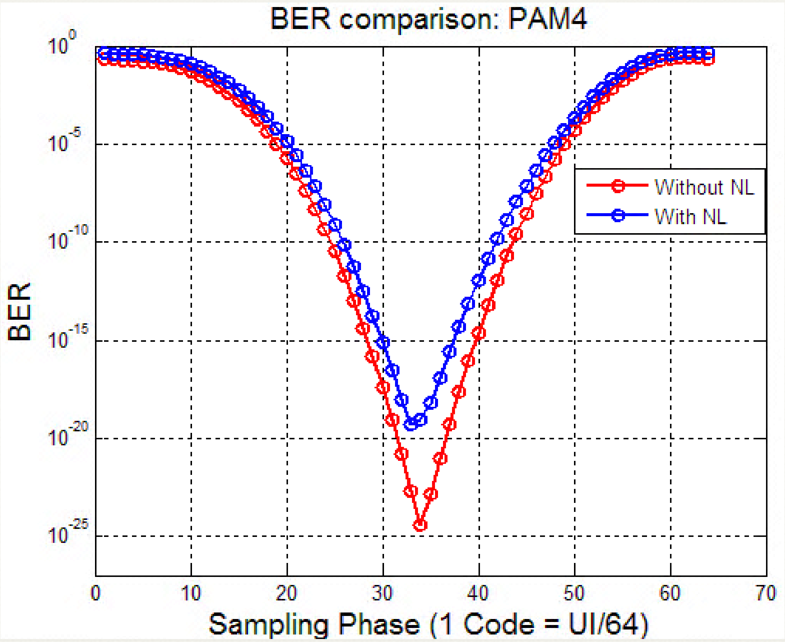
Start with the same BER,

compare the effect of NL

- Bandwidth of insertion loss, crosstalk, AWGN and CTLE for PAM2 are half that of PAM4.
- Jitter is specified as a fraction of UI, so that automatically adjusts for signaling rate.
- Since the crosstalk channel is not flat, we had to make small adjustment on gain of crosstalk channel to make the baseline BER (without NL) the same for both PAM2 & PAM4.

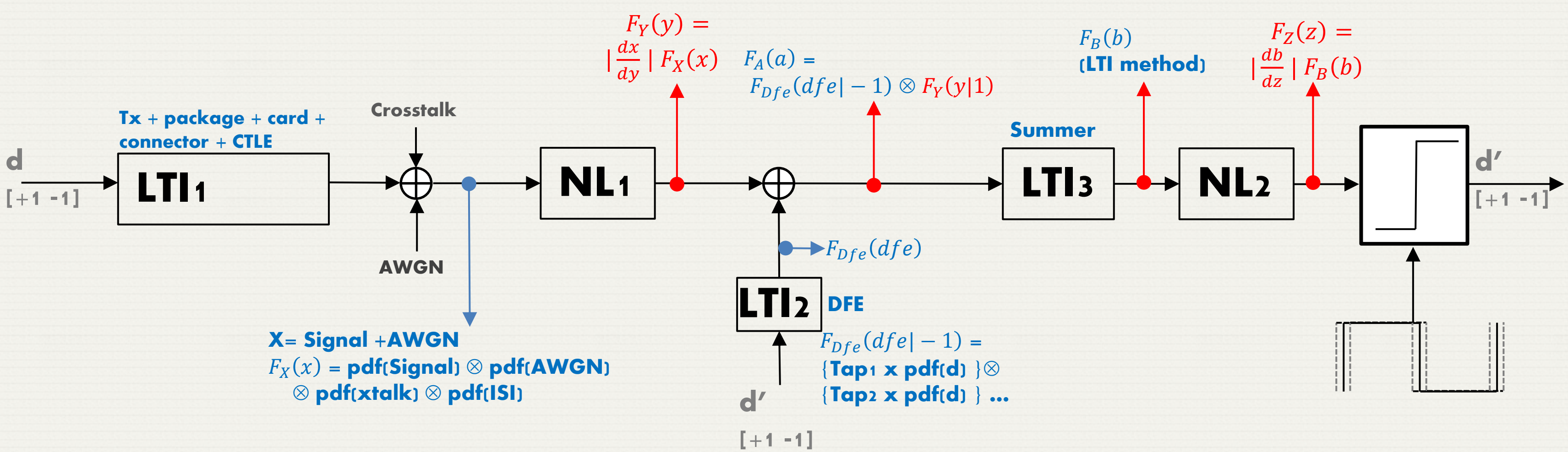
BER results: typical high speed link





	Bandwidth (Nyquist)	UI	BER without NL	BER with NL
PAM2	F_N	$1/(2^* F_N)$	1e-25	1e-23
PAM4	F_N / 2	$2/(2^* F_N)$	1e-25	1e-20

Link Model: Multiple NL blocks



PDF transformation

- Linear block : Convolution
- Nonlinear block : $F_Y(y) = \frac{F_X(x_1)}{|g'(x_1)|} + \frac{F_X(x_2)}{|g'(x_2)|} + \cdots \frac{F_X(x_n)}{|g'(x_n)|}$

Summary

- Presented methodology for calculating BER of a link in presence of nonlinearity.
  - Modification of PDF.
  - Static nonlinearity model using power series polynomial considered.
- Work ongoing to model nonlinearity using Volterra series.
- Higher order modulations are more susceptible to NL.
- Quantified the loss for a typical NL, typical high speed link.